SECTION 3. Backlash Calculations

3.00 Backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the operating pitch circles. The following section contains a description of backlash sources and a method of calculating backlash in a gear train.

3.01 Sources of Backlash
For precision trains the backlash sources are many because of the significance of every contributor even if small. There are five major descriptive groupings:

3.02 Design Backlash Allowance
1. Gear size allowance — any specific reduction of gear size (tooth thickness or testing radius) below nominal value.
2. Center distance — any specific increase in center distance above nominal value.

3.03 Major Tolerance Backlash Sources
1. Gear size to tolerance (tooth thickness or testing radius).
2. Center distance tolerance.

3.04 Gear Center Shift Due to Secondary Sources
1. Fixed-bearing eccentricities:
   a. Outer-race eccentricity of ball bearings
   b. Sleeve bearing's inside-diameter and outside-diameter runout.
2. Radial clearances due to tolerances and allowances:
   a. Ball-bearing radial play.
   b. Fit between shaft and bearing bore.
   c. Fit between bearing outside diameter and housing bore.
3. Component error sources:
   a. Clearance between component-mounting pilot diameter and housing-mounting bore.
   b. Component-mounting pilot eccentricity to shaft.
   c. Component-mounting surface flatness and perpendicularity.
   d. Component shaft radial play.

3.05 Backlash Sources Variable in Magnitude with Gear Rotation
1. Total composite error:
   a. Runout.
   b. Tooth-to-tooth errors.
   c. Lateral runout.
2. Clearance between gear bore and shaft
3. Shaft runout at point of gear mounting:
   a. Plain shafting.
   b. Stepped stud or shaft.
4. Ball-bearing rotating-race eccentricity
5. Miscellaneous runouts:
   a. Component shaft.
   b. Composite gear assembly.

3.06 Miscellaneous Sources:
1. Thermal dimensional changes.
2. Deflections: teeth, gear body, shaft, and housing.
3. Special environmental conditions — vibration, etc.

3.07 Example of Backlash Calculation
To illustrate procedures, an example of calculating gear train backlash is given. Referring to Figure 3.1, backlash from the servomotor to the antenna azimuth shaft will be calculated. This illustrates a typical problem encountered in the design of small radar antenna drive gear trains in which backlash is important to a responsive and stable servo system. Additional design conditions not given in the figure and Table 3.1 are listed below (all dimensions in inches.)

TABLE 3.1
DESIGN CONDITIONS FOR FIG. 3.1 GEAR TRAIN

Parameter	Mesh 1	Mesh 2	Mesh 3	Mesh 4
Gear size (testing radius):				
Allowance	0	0	.0005	.0005
Tolerance	+.0000 −.0007	+.0000 −.0012	+.0000 −.0015	+.0000 −.0015
Total composite error (max.)	.0005	.001	.001	.001
Center Distance:				
Allowance	0	0	0	0
Tolerance	+.001 −.000	+.002 −.000	+.002 −.000	+.002 −.000

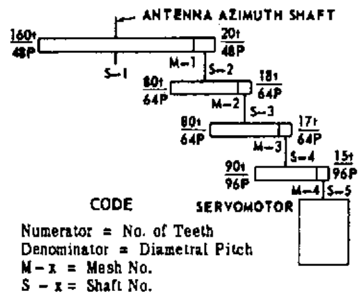

FIGURE 3.1 — GEARING SCHEMATIC AND DESIGN DETAILS FOR A SMALL RADAR ANTENNA DRIVE GEAR TRAIN

3.08 Entire TCE must be within Testing Radius Tolerance
Bearings:
All, except motor shafts, ABEC-5 ball bearings, ½ inch diameter. Radial clearance .0002 to .0006.

3.09 Housing Bore for Bearing Outside Diameter:
Allowance zero
tolerance $^{+.0003}_{-.0000}$

3.10 Shaft Diameter:
Allowance .0001 from nominal diameter
tolerance $^{+.0000}_{-.0002}$

3.11 Shaft Runout at Mounting of Gear .0002 max. TIR.

3.12 Gear Bore Diameter:
Nominal dimension $^{+.0003}_{-.0000}$

3.13 Servomotor Details:
Pinion cut on shaft — testing radius reduced .0005 under nominal, with −.0011 tolerance.

Shaft radial play .001 max.

Shaft runout .0008 max. TIR at mounting of gear.

Mounting diameter runout relative to shaft .001 TIR max.

Component-mounting design:

Mounting pilot diameter tolerance: $^{+.0000}_{-.0005}$

Mating-housing bore diameter tolerance: $^{+.0005}_{-.0000}$

Allowance: .0003.

3.14 Pinion Design:
Mesh 1 — pinned to shaft.

All other meshes — pinion is cut integral with shaft.

3.15 All Gears 20° pressure angle.

3.16 The Backlash contributors for each mesh are listed in Table 3.2. These are radial values (i.e., changes in center distance) and are converted to backlash by the factor 2 tan ϕ, in this case .728. Thus, the angular values are

$$ {}_{\theta}B_{m-i} = \frac{2\tan\phi\,\Delta C}{R_i} \times \frac{180 \times 60}{\pi} $$

Mesh 1:

$$ {}_{\theta}B_{m-1} = \frac{.728(.0052)3438}{1.6667} = 7.8 \text{ min of arc} $$

Mesh 2:

$$ {}_{\theta}B_{m-2} = \frac{.728(.0068)3438}{.6250} = 27.1 \text{ min of arc} $$

Mesh 3:

$$ {}_{\theta}B_{m-3} = \frac{.728(.0084)3438}{.6250} = 33.5 \text{ min of arc} $$

Mesh 4:

$$ {}_{\theta}B_{m-4} = \frac{.728(.00865)3438}{.4688} = 46 \text{ min of arc} $$

Summing the mesh totals, with proper velocity ratio factors relative to reference shaft S-1, the gear train maximum backlash is

$$ \begin{bmatrix} \text{Backlash from S-1 to} \\ \text{S-5 measured at S-1} \end{bmatrix} = $$

$$ B_{train} = B_{m-1} + \frac{B_{m-2}}{V_1} + \frac{B_{m-3}}{V_2} + \frac{B_{m-4}}{V_3} $$

$$ B_{train} = 7.8 + \frac{27.1}{8} + \frac{33.5}{35.6} + \frac{46}{167} $$

$$ = 12.4 \text{ min of arc} $$

Note that 63 per cent of the train backlash is in the first mesh because of the high velocity ratios of subsequent meshes. Calculation of the third mesh contribution is of questionable significance, and the fourth mesh is unnecessary.

The backlash value calculated is a maximum which will never be exceeded if parts are made according to design specification; it can be approached only if all backlash design tolerances are at their maximum values.

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TABLE 3.2
BACKLASH EXAMPLE CALCULATION

		Maximum Backlash (Radial Value)							
		Mesh 1		Mesh 2		Mesh 3		Mesh 4	
	Design Data:	Gear	Pinion	Gear	Pinion	Gear	Pinion	Gear	Component Pinion
	Shaft No. →	S-1	S-2	S-2	S-3	S-3	S-4	S-4	S-5
Backlash Source	Pitch Dia. →	3.333	.4167	1.250	.2812	1.250	.2656	.9375	.1562
Group I. Design Backlash Allowance									
*1. Center distance allowance		0	—	0	—	0	—	0	—
2. Gear size allowance		0	0	0	0	.0005	.0005	.0005	.0005
Group II. Major Tolerances									
*1. Center distance		.001	—	.002	—	.002	—	.002	—
2. Gear size		.0007	.0007	.0012	.0012	.0015	.0015	.0015	.0011
Group III. Secondary Sources									
1. Fixed-bearing eccentricities:									
a. Ball-bearing fixed race		.0001	.0001	.0001	.0001	.0001	.0001	.0001	
b. Sleeve-bearing runout									
2. Radial clearances:									
a. Ball-bearing radial play		.0003	.0003	.0003	.0003	.0003	.0003	.0003	
b. Clearance: Shaft and bearing bore									
(1) Shaft diameter tolerance		.0001	.0001	.0001	.0001	.0001	.0001	.0001	
(2) Bearing bore tolerance		.0001	.0001	.0001	.0001	.0001	.0001	.0001	
(3) Allowance		.00005	.00005	.00005	.00005	.00005	.00005	.00005	
c. Clearance: Bearing OD and housing bore									
(1) Bearing OD tolerance		.0001	.0001	.0001	.0001	.0001	.0001	.0001	
(2) Housing bore diameter tolerance		.00015	.00015	.00015	.00015	.00015	.00015	.00015	
(3) Allowance		0	0	0	0	0	0	0	
3. Component error sources:									
a. Clearance: component mounting									
(1) Component-mounting pilot dia. tolerance									.00025
(2) Housing-mounting bore dia. tolerance									.00025
(3) Allowance									.00015
b. Component's mounting pilot eccentricity									.0005
c. Component-mounting pilot flatness and perpendicularity									.0005
d. Component shaft radial play									
Group IV. Sources Variable with Rotation (one-half total value)									
1. Total composite error									
a. Runout			All TCE specified to be within testing radius tolerance						
b. Tooth-to-tooth composite									
2. Clearance: Gear mounting to shaft									
a. Gear bore diameter tolerance		.00015	.00015	.00015		.00015		.00015	
b. Shaft diameter tolerance		.0001	.0001	.0001		.0001		.0001	
c. Allowance		.00005	.00005	.00005		.00005		.00005	
3. Shaft runout at gear mounting		.0001	.0001	.0001		.0001		.0001	
4. Ball-bearing rotating-race eccentricity		.0001	.0001	.0001	.0001	.0001	.0001	.0001	
5. Miscellaneous runouts									
a. Component shaft									
b. Composite gear assembly									
6. Other sources									
Group V. Miscellaneous Sources									
1. Thermal									
2. Deflections									
3. Other sources									
	Sub-Total	.0031	.0021	.0046	.0022	.0054	.0030	.0054	.00325
	Mesh Total	.0052		.0068		.0084		.00865	

* These are values for a pair and are arbitrarily put into gear columns.